

1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics Part-2, Class XI)
Module Name/Title	Unit 7, Module 9, Bernoulli's Principle Chapter 10, Mechanical Properties Of Fluids
Module Id	Keph_201004_eContent
Pre-requisites	Conservation of energy, work energy theorem, equation of continuity, pressure in liquids, streamline flow, laminar flow, turbulent flow, Reynolds number,
Objectives	<p>After going through this lesson, the learners will be able to:</p> <ul style="list-style-type: none"> • Understand Bernoulli's theorem • Observe the decrease in pressure with increase in velocity of a flowing fluid • Explain Magnus effect • Apply Bernoulli's theorem • Cite examples of Bernoulli's theorem
Keywords	Bernoulli's theorem, Magnus effect, Torricelli law, venture meter, blood flow and heart attack

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1. UNIT SYLLABUS

UNIT 7: PROPERTIES OF BULK MATTER

Chapter-9: Mechanical Properties of Solids:

Elastic behaviour, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear, modulus of rigidity, Poisson's ratio, elastic energy.

Chapter-10: Mechanical Properties of Fluids:

Pressure due to a fluid column; Pascal's law and its applications(hydraulic lift and hydraulic brakes). Effect of gravity on fluid pressure. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its applications. Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise

Chapter-11: Thermal Properties of Matter:

Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity; C_p , C_v - calorimetry; change of state - latent heat capacity. Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wien's displacement Law, Stefan's law, Greenhouse effect.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS

17 MODULES

<p>Module 1</p>	<ul style="list-style-type: none"> ● Forces between atoms and molecules making up the bulk matter ● Reasons to believe that intermolecular and interatomic forces exist ● Overview of unit ● State of matter ● Study of a few selected properties of matter ● Study of elastic behaviour of solids ● Stationary fluid property: pressure and viscosity ● Stationary liquid property: surface tension ● Properties of Flowing fluids ● Effect of heat on matter
<p>Module 2</p>	<ul style="list-style-type: none"> ● Idea of deformation by external force ● Elastic nature of materials ● Elastic behaviour ● Plastic behaviour ● Tensile stress ● Longitudinal Stress and longitudinal strain ● Relation between stress and strain ● Hooke's law ● Young's modulus of elasticity 'Y'
<p>Module 3</p>	<ul style="list-style-type: none"> ● Searle's apparatus ● Experiment to determine Young's modulus of the material of a wire in the laboratory ● What do we learn from the experiment?
<p>Module 4</p>	<ul style="list-style-type: none"> ● Volumetric strain ● Volumetric stress ● Hydraulic stress ● Bulk modulus K ● Fish, aquatic life on seabed, deep sea diver suits and submarines
<p>Module 5</p>	<ul style="list-style-type: none"> ● Shear strain ● Shear stress ● Modulus of Rigidity G ● Poisson's ratio ● Elastic energy ● To study the effect of load on depression of a suitably clamped meter scale loaded at i) its ends ii) in the middle ● Height of sand heaps, height of mountains
<p>Module 6</p>	<ul style="list-style-type: none"> ● Fluids-liquids and gases

	<ul style="list-style-type: none"> • Stationary and flowing fluids • Pressure due to a fluid column • Pressure exerted by solid , liquids and gases • Direction of Pressure exerted by solids, liquids and gases
Module 7	<ul style="list-style-type: none"> • Viscosity- coefficient of viscosity • Stokes' Law • Terminal velocity • Examples • Determine the coefficient of viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body in the laboratory
Module 8	<ul style="list-style-type: none"> • Streamline and turbulent flow • Critical velocity • Reynolds number • Obtaining the Reynolds number formula using method of dimensions • Need for Reynolds number and factors effecting its value • Equation of continuity for fluid flow • Examples
Module 9	<ul style="list-style-type: none"> • Bernoulli's theorem • To observe the decrease in pressure with increase in velocity of a fluid • Magnus effect • Applications of Bernoulli's theorem • Examples • Doppler test for blockage in arteries
Module 10	<ul style="list-style-type: none"> • Liquid surface • Surface energy • Surface tension defined through force and through energy • Angle of contact • Measuring surface tension
Module 11	<ul style="list-style-type: none"> • Effects of surface tension in daily life • Excess pressure across a curved liquid surface • Application of surface tension to drops, bubbles • Capillarity • Determination of surface tension of water by capillary rise method in the laboratory • To study the effect of detergent on surface tension of water through observations on capillary rise.
Module 12	<ul style="list-style-type: none"> • Thermal properties of matter • Heat

	<ul style="list-style-type: none"> • Temperature • Thermometers
Module 13	<ul style="list-style-type: none"> • Thermal expansion • To observe and explain the effect of heating on a bi-metallic strip • Practical applications of bimetallic strips • Expansion of solids, liquids and gases • To note the change in the level of liquid in a container on heating and to interpret the results • Anomalous expansion of water
Module 14	<ul style="list-style-type: none"> • Rise in temperature • Heat capacity of a body • Specific heat capacity of a material • Calorimetry • To determine specific heat capacity of a given solid material by the method of mixtures • Heat capacities of a gas have a large range • Specific heat at constant volume C_v • Specific heat capacity at constant pressure C_p
Module 15	<ul style="list-style-type: none"> • Change of state • To observe change of state and plot a cooling curve for molten wax. • Melting point, Regelation, Evaporation, boiling point, sublimation • Triple point of water • Latent heat of fusion • Latent heat of vaporisation • Calorimetry and determination of specific latent heat capacity
Module 16	<ul style="list-style-type: none"> • Heat Transfer • Conduction, convection, radiation • Coefficient of thermal conductivity • Convection
Module 17	<ul style="list-style-type: none"> • Black body • Black body radiation • Wien's displacement law • Stefan's law • Newton's law of cooling, • To study the temperature, time relation for a hot body by plotting its cooling curve • To study the factors affecting the rate of loss of heat of a liquid • Greenhouse effect

MODULE 9

3. WORDS YOU MUST KNOW

Pressure: Force per unit area, or thrust per unit area.

Fluid: Any material capable of flow.

Fluid pressure: ρgh

Up thrust: The upward pressure on an object wholly or partially immersed in a fluid at rest = weight of fluid displaced by the solid object.

Atmospheric pressure: The pressure of atmosphere at any point is equal to the weight of column of air of unit cross sectional area extending from that point to the top of the atmosphere. At sea level it is 1.013×10^5 Pa (1 atm).

Gauge pressure: The pressure P , at depth below the surface of a liquid open to the atmosphere is greater than the atmospheric pressure by an amount ρgh . The excess of pressure, $P - P_a$, at depth h is called the gauge pressure at that point.

Manometer: An instrument for measuring pressure differences.

Pascal's law for transmission of pressure: Whenever external pressure is applied on any part of a fluid contained in a vessel, it is transmitted undiminished and equally in all directions.

Hydraulic pressure: Fluid pressure on any object immersed fully in a fluid.

Steady flow: The flow of the fluid is said to be steady if at any given point, the velocity of each passing fluid particle remains constant in time.

Streamline: The path taken by a fluid particle under a steady flow is a streamline.

Stream line flow: Steady flow of a fluid is called stream line flow.

Turbulent flow: Steady flow is achieved at low flow speeds. Beyond a limiting value of speed, called critical speed, this flow loses steadiness and becomes turbulent.

Reynolds number: A number which indicates whether the flow is streamline or turbulent.

Equation of continuity for fluid flow: In steady flow the volume per second of fluid passing a point remains the same.

4. INTRODUCTION

Fluid flow is a complex phenomenon. But we can obtain some useful properties for steady or streamline flows using the conservation of energy.

A slow-moving fluid exerts more pressure than a fast-moving fluid on the walls of the pipe or tube or drain

Since "fluid" in this context applies equally to liquids and gases, the principle has as many applications with regard to airflow as to the flow of liquids.

One of the most dramatic everyday examples of Bernoulli's principle can be found in an airplane, which stays aloft due to pressure difference on the surfaces of its wing; but the truth of the principle is also illustrated in something as gruesome as a temporary roof flying off on a windy day.

DO IT YOURSELF

Hold a paper strip say 3 cm x 30 cm

Blow on top the paper lifts up

Why?

Bernoulli's Principle Demo

Hold a sheet of paper in front of your mouth and blow; the paper will rise.



<https://www.youtube.com/watch?v=krC1QYZLWu0>

The Swiss mathematician and physicist Daniel Bernoulli (1700-1782) discovered the principle that bears his name while conducting experiments concerning an even more fundamental concept: the conservation of energy.



This is a law of physics that holds that a system isolated from all outside influences maintains the same total amount of energy, though energy transformations from one form to another may take place.

He conducted experiments on the conservation of energy using liquids, observing how water flows through pipes of varying diameter. In a segment of pipe with a relatively

large diameter, he observed, water flowed slowly, but as it entered a segment of smaller diameter, its speed increased.



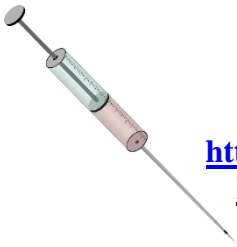
https://cdn.pixabay.com/photo/2015/01/05/03/13/irrigation-588941_960_720.jpg

The misty spread of water is due to Bernoulli's principle

this idea is used in sprinklers, the nozzle through which the water jets emerge out with pressure is due to fact that the nozzle head diameter is smaller than the rest of the pipe.

Look out for such sprinklers in the parks and gardens and observe them carefully

Have you a doctor's syringe. there is a wide cylinder with a plunger. A thin needle is attached to it .The doctor uses the plunger to fill the syringe and to give us the medicine.



<https://www.needpix.com/photo/22739/injection-syringe-needle-hypodermic-shot-inject-drug-healthcare-vaccination>

when the plunger is pushed in a filled syringe the medicine flows . the area of cross section ,through which the medicine flows is different . it is more in the cylindrical part and very narrow inside the needle.

since the width of the narrower needle is smaller, the fluid must move faster, for continuous flow

Another way to illustrate this is to observe the behaviour of a river: in a wide, un constricted region, it flows slowly, but if it is made to flow in a narrow manmade embankments (for instance),. then it speeds up dramatically.



https://cdn.pixabay.com/photo/2015/10/12/14/54/river-983958_960_720.jpg



http://katemorganimages.com/wp/wp-content/uploads/galleries/post-280/WaterRocks_19x12.jpg

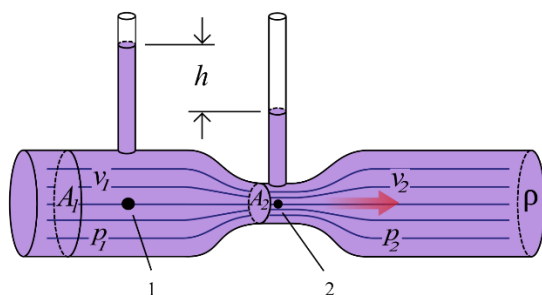
boulders create narrow paths for the flow hence water flows faster.

The above is a result of the fact that water, being a fluid, adjusts its shape according to obstructions in its path..

Since for continuous steady flow, the volume of fluid flowing per second passing every section of the pipe will be the same,

Logically ,there must be a decrease in pressure with increase in speed. Hence **Bernoulli's conclusion: the slower the rate of flow, the higher the pressure, and the faster the rate of flow, the lower the pressure.**

Recall equation of continuity



<https://upload.wikimedia.org/wikipedia/commons/thumb/4/4b/Venturi5.svg/2000px-Venturi5.svg.png>

The density of a fluid can be considered to be constant and .the fluid can be considered to be incompressible.

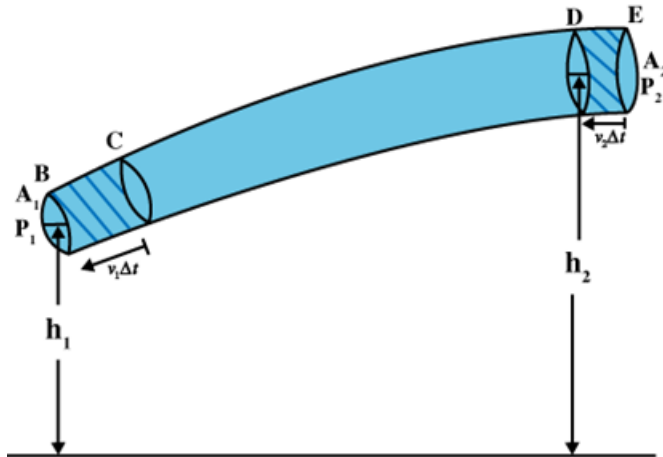
So, incompressible fluids have to speed up when they reach a narrow constricted section in order to maintain a constant volume flow rate. This is why a narrow nozzles are used on a hose pipe ,to water the garden quickly.

Within a horizontal flow of fluid, points of higher fluid speed will have less pressure than points of slower fluid speed.

The rise of fluid in a pipe attached to a region of smaller area of cross section is smaller as compared to a wider portion of the pipe.

5. BERNOULLI'S PRINCIPLE

Consider a fluid moving in a pipe of varying cross-sectional area. Let the pipe be at varying heights as shown in the figure



The flow of an ideal fluid in a pipe of varying cross section. The fluid in a section of length $v_1\Delta t$ to the section of length $v_2\Delta t$

We now suppose that an **incompressible fluid** is flowing through the pipe in a **steady flow**. Its velocity must change according to the equation of continuity.

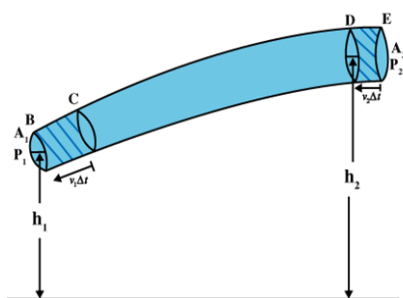
Logically **A force is required to produce the change in velocity or cause acceleration, which due to the fluid surrounding it, hence, the pressure must be different in different regions.** This can be understood as, for a fluid to flow there must be a pressure difference.

Bernoulli's equation is a general expression that relates the pressure difference between two points in a pipe to both velocity changes (kinetic energy change) and elevation (height) changes (potential energy change).

It can therefore be considered as a case of work energy conversion

Swiss Physicist Daniel Bernoulli established a relation between work done on a fluid due to pressure difference and the change in its energy (both kinetic as well as potential energies) in 1738.

6. BERNOULLI'S EQUATION



Consider the flow at two regions 1 (i.e. BC) and 2 (i.e. DE).

Consider the fluid initially lying between B and D. In an infinitesimal time interval Δt , this fluid would have moved.

Suppose v_1 is the speed at B and v_2 at D, then fluid initially at B has moved a distance to C ($v_1\Delta t$ is small enough to assume constant cross-section along BC).

In the same interval Δt the fluid initially at D moves to E, a distance equal to $v_2\Delta t$.

Pressures P_1 and P_2 act as shown on the plane faces of areas A_1 and A_2 , connecting the two regions.

Considering work energy theorem, we can relate the work done in moving the fluid to the change in kinetic and potential energy. This will take care of dissimilar area of cross section and dissimilar height of the pipe with respect to a reference horizontal level.

The work done on the fluid at left end (BC) is

$$W_1 = F_1 \cdot ds$$

$$F_1 = P_1 A_1 \text{ and } ds = \text{speed} \times \text{time} = v_1 \Delta t$$

$$W_1 = P_1 A_1 (v_1 \Delta t) = P_1 \Delta V$$

Here $\Delta V =$ volume of fluid flowing past, point 1 in time $\Delta t = A_1 (v_1 \Delta t)$

NOTICE

$A_1 =$ area of cross section at 1

$v_1 =$ velocity of fluid at 1

$\Delta V =$ volume of fluid passing 1 in time Δt

Since the same volume ΔV passes through both the regions (from the equation of continuity)

The work done by the fluid at the other end (DE) is

$$W_2 = P_2 A_2 (v_2 \Delta t) = P_2 \Delta V \text{ or,}$$

The work done on the fluid is $-P_2 \Delta V$

So the total work done on the fluid is $W_1 - W_2 = (P_1 - P_2) \Delta V$

Now, Part of this work goes into changing the kinetic energy of the fluid, and part goes into changing the gravitational potential energy.

If the density of the fluid is ρ and

$$\Delta m = \rho A_1 v_1 \Delta t = \rho \Delta V$$

is the mass passing through the pipe in time Δt ,

Then

Change in **gravitational potential energy** is ΔU

$$= \rho g \Delta V (h_2 - h_1)$$

The change in **its kinetic energy** is ΔK

$$= \left(\frac{1}{2}\right) \rho \Delta V (v_2^2 - v_1^2)$$

We can employ the work – energy theorem to this volume of the fluid and this yields

$$(P_1 - P_2)(\Delta V) = \rho g \Delta V (h_2 - h_1) + \left(\frac{1}{2}\right) \rho \Delta V (v_2^2 - v_1^2)$$

We now divide each term by ΔV to obtain

$$(P_1 - P_2) = \rho g (h_2 - h_1) + \left(\frac{1}{2}\right) \rho (v_2^2 - v_1^2)$$

We can rearrange the above terms to obtain

$$P_1 + \rho g h_1 + \left(\frac{1}{2}\right) \rho (v_1^2) = P_2 + \rho g h_2 + \left(\frac{1}{2}\right) \rho (v_2^2)$$

This is Bernoulli's equation.

Since 1 and 2 refer to any two locations along the pipe, we may write the expression in general as

$$P + \left(\frac{1}{2}\right) \rho (v^2) + \rho g h = \text{constant}$$

In words, the Bernoulli's relation may be stated as follows:

Along a streamline the sum of the pressure (P), the kinetic energy per unit volume $\left(\frac{1}{2}\right) \rho (v^2)$ and the potential energy per unit volume ($\rho g h$) remains a constant.

Note

In applying the energy conservation principle, we have to keep in mind

- **There is an assumption that no energy is lost due to friction.**

But, when fluids flow, some energy does get lost due to internal friction. This arises due to the fact that in a steady fluid flow, the different layers of the fluid flow with different velocities.

These layers exert frictional forces on each other resulting in a loss of energy. This property of the fluid is called viscosity and is discussed in more detail in earlier module. The lost kinetic energy of the fluid gets converted into heat energy.

- **Bernoulli's equation ideally applies to fluids with zero viscosity or non-viscous fluids.** Which is really non-existent
- **Another restriction on application of Bernoulli theorem is that the fluids must be incompressible,** as the elastic energy of the fluid is also not taken into consideration.

In practice, it has a large number of useful applications and can help explain a wide variety of phenomena for **low viscosity incompressible fluids.**

- Bernoulli's equation also does not hold for non-steady or turbulent flows, because in that situation velocity and pressure are constantly changing.
- **When a fluid is at rest** i.e. its velocity of flow, is zero everywhere, Bernoulli's equation becomes

$$(P_1 - P_2) = \rho g(h_2 - h_1)$$

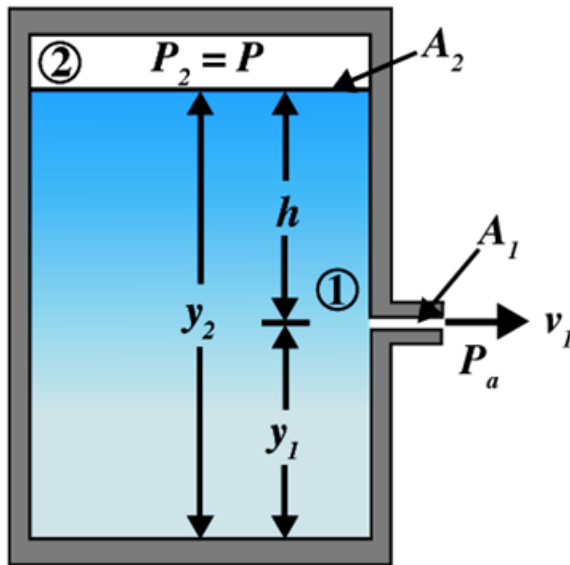
Watch animation on

https://www.vascak.cz/data/android/physicsatschool/templateimg.php?s=mech_bernoulli&l=en

7. SPEED OF EFFLUX: TORRICELLI'S LAW

The word efflux means fluid outflow. **Torricelli** discovered that the speed of efflux from tank is given by a formula identical to that of a freely falling body.

Consider a tank containing a liquid of density ρ with a small hole in its side at a height y_1 from the bottom



Torricelli's law: The speed of efflux, v_1 , from the side of the container is given by the application of Bernoulli's equation. If the container is open at the top to the atmosphere then $v_1 = \sqrt{2gh}$

The air above the liquid, whose surface is at height y_2 , is at pressure P and has an area of cross section A_2 . From the equation of continuity

we have

$$v_1 A_1 = v_2 A_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$

If the cross sectional area of the tank A_2 is much larger than that of the hole ($A_2 \gg A_1$), then

We may take the fluid to be approximately at rest at the top, i.e. $v_2 = 0$. Now applying the

Bernoulli equation at points 1 and 2 and noting that at the hole $P_1 = P_a$, the atmospheric pressure, we have from

$$P_a + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

Taking $y_2 - y_1 = h$ we have

$$v_1 = \sqrt{2gh + \frac{2(P - P_a)}{\rho}}$$

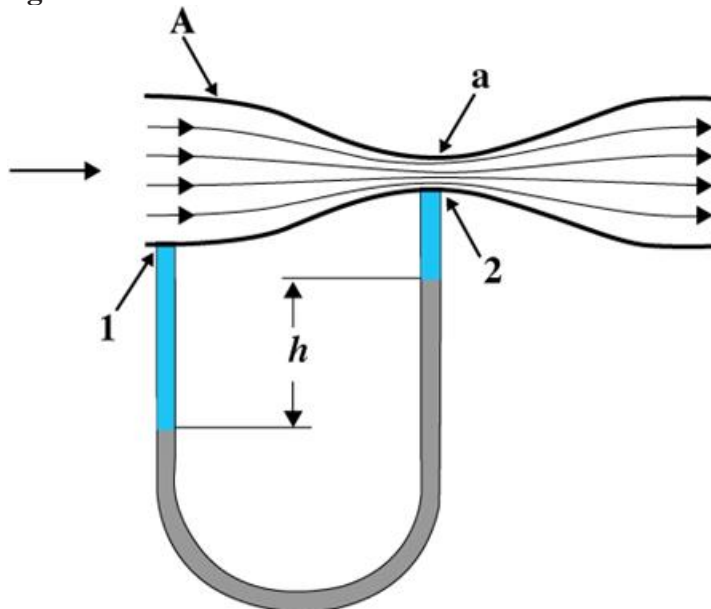
When $P \gg P_a$ and $2gh$ may be ignored, the speed of efflux is determined by the container pressure.

Such a situation occurs in rocket propulsion. On the other hand if the tank is open to the atmosphere, then $P = P_a$ and $v = \sqrt{2gh}$

This is the speed of a freely falling body is known as **Torricelli's law**.

8. VENTURI-METER

The Venturi-meter is a device to measure the flow speed of incompressible fluid. It consists of a tube with a broad diameter and a small constriction at the middle as shown in the figure



A manometer in the form of a U-tube is also attached to it, with one arm at the broad neck point of the tube and the other at constriction as shown.

The manometer contains a liquid of density ρ_m . The speed v_1 of the liquid flowing through the tube at the broad neck area A is to be measured from equation of continuity the

Speed at the constriction becomes

$$v_2 = \frac{A}{a} v_1$$

Then using Bernoulli's equation, we get

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

So that

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2 \left\{ \left(\frac{A}{a} \right)^2 - 1 \right\}$$

Also

$$P_1 - P_2 = \rho_m g h = \frac{1}{2}\rho v_1^2 \left\{ \left(\frac{A}{a} \right)^2 - 1 \right\}$$

So that the speed of fluid at wide neck is

$$v_1 = \sqrt{\left(\frac{2\rho_m g h}{\rho} \right) \left(\left(\frac{A}{a} \right)^2 - 1 \right)^{-1/2}}$$

This pressure difference causes the fluid in the U tube connected at the broad neck to rise in comparison to the other arm.

The difference in height h measure the pressure difference.

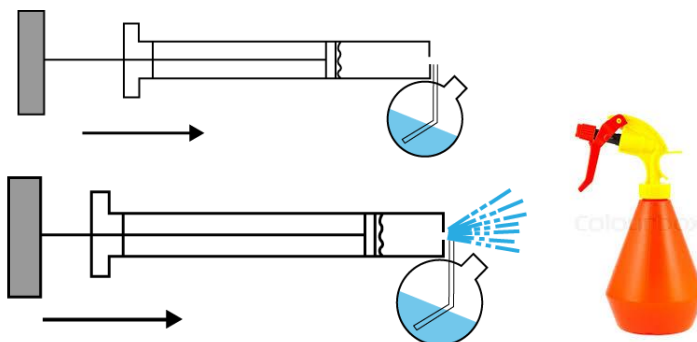
The principle behind this meter has many applications.

The carburettor of automobile has a Venturi channel (nozzle) through which air flows with a large speed. The pressure is then lowered at the narrow neck and the petrol is sucked up in the chamber to provide the correct mixture of air to fuel necessary for combustion.

Filter pumps or aspirators, Bunsen burners in the chemistry laboratories, atomisers and sprayers used for perfumes or to spray insecticides work on the same principle.

Working of a spray gun

The spray gun. Piston forces air at high speeds causing a lowering of pressure at the neck of the container



https://d2gg9evh47fn9z.cloudfront.net/800px_COLOURBOX2616602.jpg

Activities you can do



<https://www.youtube.com/watch?v=ZRSfxUviAlg>

This is a wonderful experiment showing Bernoulli's Theorem. We will need two paper cups, knife, two straws, scissors and a balloon. First we will make a hole in the base of each paper cup. Then we will press fit a bendable straw in the holes of both cups. Then we will make a few slits on the surface of one paper cup so that the air blown through the straw can pass through. Then we will inflate the balloon to half. Now we will have two cups with straws. One cup will have slits the other will have no slits. Now we will place the inflated balloon in the cup without the slit and blow through the straw. No matter how hard we blow the balloon does not fall. We can also invert with the balloon down. On blowing the unsupported balloon will also not fall. We can test this again and again. Now repeat the same experiment using the cup with the slits. On blowing through the straw the balloon will rise in the air. Why does this happen? On blowing through the plain cup air comes out at a high velocity and creates a low-pressure zone sucking the balloon on the cup. On the other hand when air is blown in the cup with the slits most of the air escaping through the slits is at atmospheric pressure and some of the air goes up lifting the balloon. This is a wonderful way to illustrate Bernoulli principle.



https://www.youtube.com/watch?v=VyZ_sjtjyrM

The Bernoulli Cone is a very exciting experiment. For this you will need a piece of newspaper, a cardboard reel and some tape. Fold the A-4 size sheet of newspaper into a cone. Tape the conical end with tape. Cut off the extra paper from the flared end to make a regular cone. Now nip the pointed end of the cone with scissors to make a hole. The hole should be just big enough so that a cardboard thread reel can go snugly into it. Place the reel in the newspaper funnel. The apparatus is ready to perform the experiment. Now when you blow through the reel air comes out at a high speed and the

two layers of the newspaper come and hit each other. This is because of Bernoulli's theorem which states that high velocity creates a low pressure zone which collapses the cone.



Bernoulli Bag

<https://www.youtube.com/watch?v=1TbTS7CWe4Q>

Watch the video and explain



Flying Tape

<https://www.youtube.com/watch?v=sRfTYP-XVhU>

The air flow above the tape produces low pressure that balances its weight

EXAMPLE

The flow of blood in a large artery of an anaesthetised dog is diverted through a Venturi meter. The wider part of the meter has a cross sectional area equal to that of the artery. $A = 8 \text{ mm}^2$. The narrower part has an area $a = 4 \text{ mm}^2$. The pressure drop in the artery is 24 Pa.

What is the speed of the blood in the artery?

SOLUTION

We take the density of blood to be $1.06 \times 10^3 \text{ kg m}^{-3}$.

The ratio of the areas is $\left(\frac{A}{a}\right) = 2$. using this equation

$$v_1 = \sqrt{\left(\frac{2\rho mgh}{\rho}\right) \left(\left(\frac{A}{a}\right)^2 - 1\right)^{-1/2}}$$

we obtain,

$$v_1 = \sqrt{\frac{2 \times 24 \text{ Pa}}{1060 \text{ kg m}^{-3} \times (2^2 - 1)}} = 0.123 \text{ ms}^{-1}$$

EXAMPLE

Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1 cm. If the amount of glycerine collected per second at one end is $4.0 \times 10^{-3} \text{ kg s}^{-1}$, what is the pressure difference between the two ends of the tube?

Given that density of glycerine = $1.3 \times 10^3 \text{ kg m}^{-3}$ and viscosity of glycerine = 0.83 Pa s

SOLUTION:

Here, $l = 1.5 \text{ m}$; $r = 1 \text{ cm} = 0.01 \text{ m}$; $\eta = 0.83 \text{ Pa s}$

Mass of the glycerine flowing per second,

$$m = 4.0 \times 10^{-3} \text{ kg s}^{-1}$$

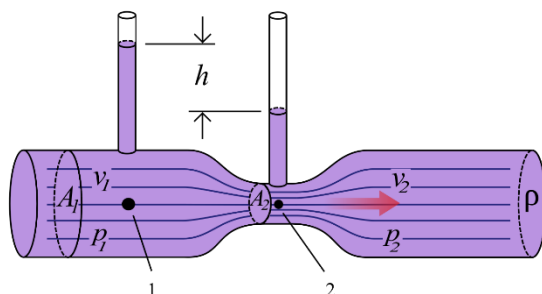
Therefore, volume of glycerine flowing per second,

$$V = \frac{m}{\rho} = \frac{4.0 \times 10^{-3}}{1.3 \times 10^3} = 3.077 \times 10^{-6} \text{ m}^3 \text{ s}^{-1}$$

Now, if $V = \frac{\pi p r^4}{8 \eta l}$

$$\text{Or } p = \frac{8V\eta l}{\pi r^4} = \frac{8 \times 3.077 \times 10^{-6} \times 0.83 \times 1.5}{\pi \times (0.01)^4} = 975.52 \text{ Pa}$$

EXAMPLE



Observe the water levels in the two vertical pipes shown.

a) Is it possible to compare the

- i) velocities at 1 and 2
- ii) Diameter of pipe at 1 and 2

- b) Can the velocity of blood flow in different sections of the blood vessels show blockages?
- c) What is the echo test that we hear about for carotid artery or heart?

SOLUTION

Hint

a) i) $v_1 = \sqrt{\left(\frac{2\rho_w gh}{\rho}\right) \left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)^{-1/2}}$
 ii) Diameter = $2\sqrt{\frac{A}{\pi}}$

b) yes

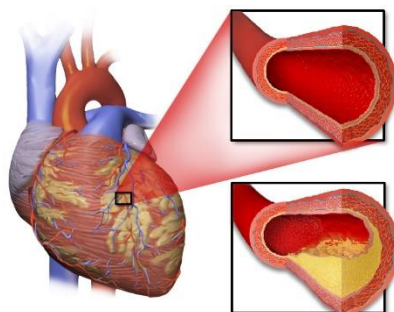
c) Echo test/ultra sound is a diagnostic test to determine blockages in carotid artery and heart. It uses sound waves and Doppler Effect to find the velocity of blood flow.

In an **ultrasound** examination, a device/probe, both sends the sound waves into the body and receives the echoing waves.

When the transducer/device is pressed against the skin, it directs small pulses of inaudible, high-frequency sound waves into the body.

Doppler mode exploits the frequency shift due to relative motion of blood. With this information regarding blood velocity, blockages in arteries and cardiac valves are diagnosed.

9. BLOOD FLOW AND HEART ATTACK(just for understanding)



Normal and Partially Blocked Blood Vessels

Bernoulli's principle helps in explaining blood flow in an artery. The artery may get constricted due to the accumulation of plaque on its inner walls. In order to drive the blood through this constriction a greater demand is placed on the activity of the heart.

The speed of the flow of the blood in this region is raised which lowers the pressure inside and the artery may collapse due to the external pressure. The heart exerts further pressure to open this artery and forces the blood through. As the blood rushes through the opening, the internal pressure once again drops due to same reasons leading to a repeat collapse. This may result in heart attack.

10. DYNAMIC LIFT AND MAGNUS EFFECT

Dynamic lift is the force that acts on a body, such as an airplane wing, a hydrofoil or a spinning ball, by virtue of its motion through a fluid. In many games such as cricket, tennis, baseball, or golf, we notice that a spinning ball deviates from its parabolic trajectory as it moves through air. This deviation can be partly explained on the basis of Bernoulli's principle.

(i) Ball moving without spin:

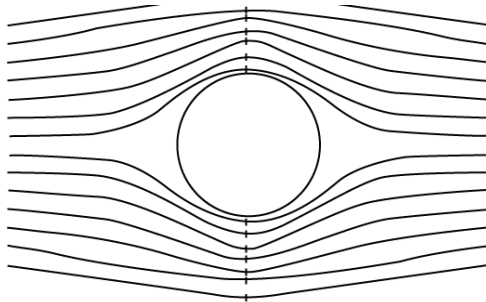
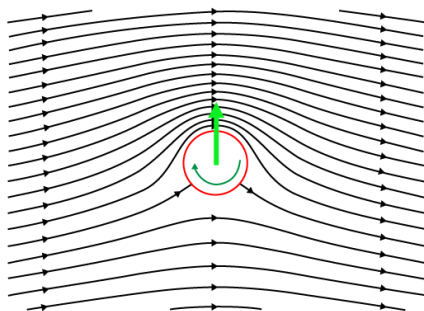


Fig shows the streamlines around a non-spinning ball moving relative to a fluid.

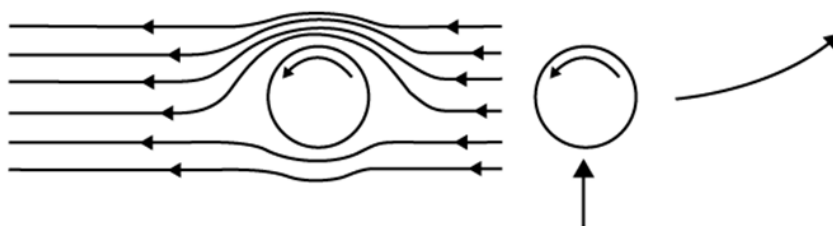
From the symmetry of streamlines, it is clear that the velocity of fluid (air) above and below the ball at corresponding points is the same resulting in zero pressure difference.

The air therefore, exerts no upward or downward force on the ball.

(ii) Ball moving with spin:



https://upload.wikimedia.org/wikipedia/commons/thumb/8/8d/Streamline_of_magnus_effect.svg/2000px-Streamline_of_magnus_effect.svg.png



[https://upload.wikimedia.org/wikipedia/commons/thumb/f/f7/PSM_V83_D205_Bernoulli principle applied to curving of baseball 3.png/320px-PSM_V83_D205_Bernoulli principle applied to curving of baseball 3.png](https://upload.wikimedia.org/wikipedia/commons/thumb/f/f7/PSM_V83_D205_Bernoulli_principle_applied_to_curving_of_baseball_3.png/320px-PSM_V83_D205_Bernoulli_principle_applied_to_curving_of_baseball_3.png)

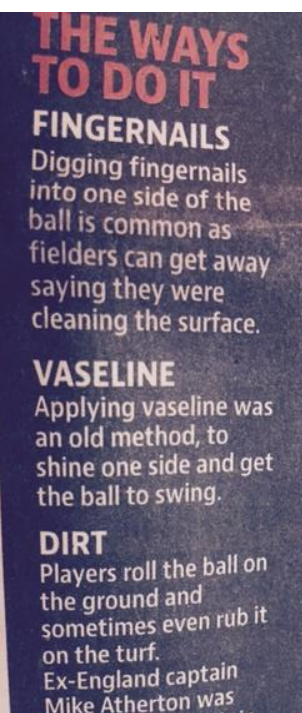
A spinning ball drags air along with it. If the surface is rough more air will be dragged. The figure shows the streamlines of air for a ball which is moving and spinning at the same time. The ball is moving forward and relative to it the air is moving backwards.

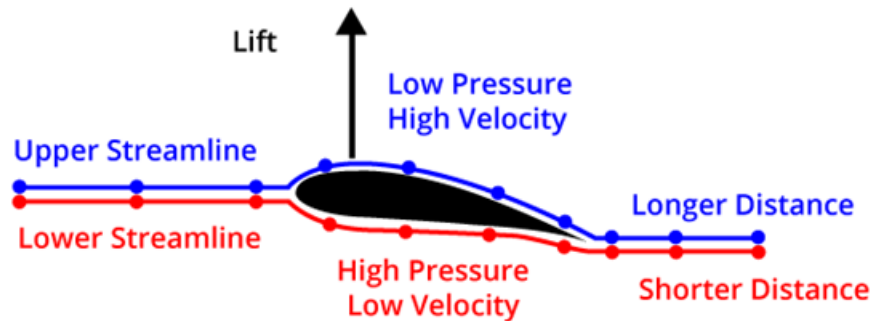
Therefore, the velocity of air above the ball relative to it is larger and below it is smaller. The stream lines thus get crowded above and rarefied below. This difference in the velocities of air results in the pressure difference between the lower and upper faces and there is a net upward force on the ball. This dynamic lift due to spinning is called **Magnus effect**.

In cricket matches bowlers often tamper with the ball to make the swing according to their choice, by making part of the surface of the ball rougher than the other. This confuses the batsman and the choice of strike stroke resulting in bowler getting the better of the batsman. This is not acceptable behaviour of sportspersons and is not appreciated. The recent news about ball tampering was covered in various sections of the media and is widely known. Pictures of the newspaper cutting are given here.



SAD BUT TRUE



iii) Aerofoil or lift on aircraft wing:


The figure shows an aerofoil, which is a solid piece shaped to provide an upward dynamic lift when it moves horizontally through air.

The cross section of the wings of an aeroplane looks somewhat like the aerofoil with streamlines around it. When the aerofoil moves against the wind, the orientation of the wing relative to flow direction causes the streamlines to crowd together above the wing more than those below it. This is on account of its shape. The flow speed on top is higher than that below it.

A pressure difference so created, produces an upward force resulting in a dynamic lift of the wings and this balances the weight of the plane along with passenger's luggage and all

EXAMPLE

A fully loaded Boeing aircraft has a mass of $3.3 \times 10^5 \text{ kg}$. Its total wing area is 500 m^2 . It is in level flight with a speed of 960 km/h .

- (a) Estimate the pressure difference between the lower and upper surfaces of the wings
 (b) Estimate the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface.

[The density of air is $\rho = 1.2 \text{ kg m}^{-3}$]

SOLUTION

- a) The weight of the Boeing aircraft is balanced by the upward force due to the Pressure difference

$$\begin{aligned} \Delta P \times A &= 3.3 \times 10^5 \text{ kg} \times 9.8 \text{ m s}^{-2} \\ \Delta P &= 3.3 \times 10^5 \text{ kg} \times 9.8 \text{ m s}^{-2} / 500 \text{ m}^2 \\ &= 6.5 \times 10^3 \text{ Nm}^{-2} \end{aligned}$$

- (b) We ignore the small height difference between the top and bottom sides and use Bernoulli's theorem. The pressure difference between the top and bottom is then, where v_2 is the speed of air over the upper surface and v_1 is the speed under the bottom surface,

$$\Delta P = \frac{\rho}{2} (v_2^2 - v_1^2)$$

or, $(v_2 - v_1) = \frac{2\Delta P}{\rho(v_2 + v_1)}$

Taking the average speed

$$v_{\text{av}} = \frac{v_2 + v_1}{2} = 960 \frac{\text{km}}{\text{h}} = 267 \text{ m s}^{-1}$$

We have fractional increase in speed:

$$(v_2 - v_1)/v_{av} = \frac{\Delta P}{\rho v_{av}^2} \approx 0.08$$

The speed above the wing needs to be only 8 % higher than that below

11. SUMMARY

- **The volume of an incompressible fluid passing any point every second in a pipe of non-uniform cross section is the same in the steady flow.**
 $v A = \text{constant}$ (v is the velocity and A is the area of cross section)
 The equation is due to mass conservation in incompressible fluid flow.
- **Bernoulli's principle states that as we move along a streamline, the sum of the pressure (P), the kinetic energy per unit volume $\rho v^2/2$ and the potential energy per unit volume (ρgh) remains a constant.**

$$P + \frac{\rho v^2}{2} + \rho gh = \text{constant}$$
- **The equation is basically the conservation of energy applied to non-viscous fluid motion in steady state. There is no fluid which have zero viscosity, so the above statement is true only approximately. The viscosity is like friction and converts the kinetic energy to heat energy.**
- **Bernoulli's principal does not hold in presence of viscous drag on the fluid. The work done by this dissipative viscous force must be taken into account in this case, and P_2 will be lower than the value given by Bernoulli's equation.**